The FitzHugh-Nagumo model

Analog computers already played a large role in the early days of modelling the behavior of neurons.[1] This model was inspired by the rather complex HODGKIN-HUXLEY model[2] and is described by

$$\begin{aligned}\dot{V} &= V - \frac{V^3}{3} - W + 2I_{\text{ext}}\\ \dot{W} &= \tau^{-1}(V + a - bW)\end{aligned} \tag{1}$$

with $a = 0.7$, $b = 0.8$, and $\tau = 12.5$.[3] The variable V denotes the membrane potential while W describes a recovery variable. I_{ext} is a stimulus current applied externally to the cell.[4] This value is the single input to the neuron and controls its behavior.

Scaling this system is straightforward, starting with the scaling factor $\lambda_v = \frac{1}{2}$, yielding

$$\begin{aligned}\dot{\widehat{V}} &= \lambda_V \left(2\widehat{V} - \frac{8}{3}\widehat{V}^3 - W + 2I_{\text{ext}}\right)\\ &= \widehat{V} - \frac{4}{3}\widehat{V}^3 - \frac{1}{2}W + I_{\text{ext}}.\end{aligned} \tag{2}$$

Of course, λ_V must be compensated for in equation 1 yielding

$$\dot{W} = 2\tau^{-1}\widehat{V} + \tau^{-1}a - \tau^{-1}bW$$

in turn.

A suitable scaling factor for W is $\lambda_W = \frac{1}{3}$ yielding

$$\begin{aligned}\dot{\widehat{W}} &= \lambda_W \left(2\tau^{-1}\widehat{V} + \tau^{-1}a - 3\tau^{-1}b\widehat{W}\right)\\ &= \frac{2}{3}\tau^{-1}\widehat{V} + \frac{1}{3}\tau^{-1}a - \tau^{-1}b\widehat{W}.\end{aligned}$$

[1] See [IZHIKEVICH et al. 2026]. In fact, the stories told by FITZHUGH about the temperamental analog computer beasts of his time are quite funny.

[2] See [RAMAN et al. 2021] (this book has quickly become one of the author's favorite books).

[3] See [IZHIKEVICH et al. 2026] and [IZHIKEVICH 2007, pp. 106 ff.].

[4] The factor 2 in front of this term has been chosen for convenience and will cancel out soon.

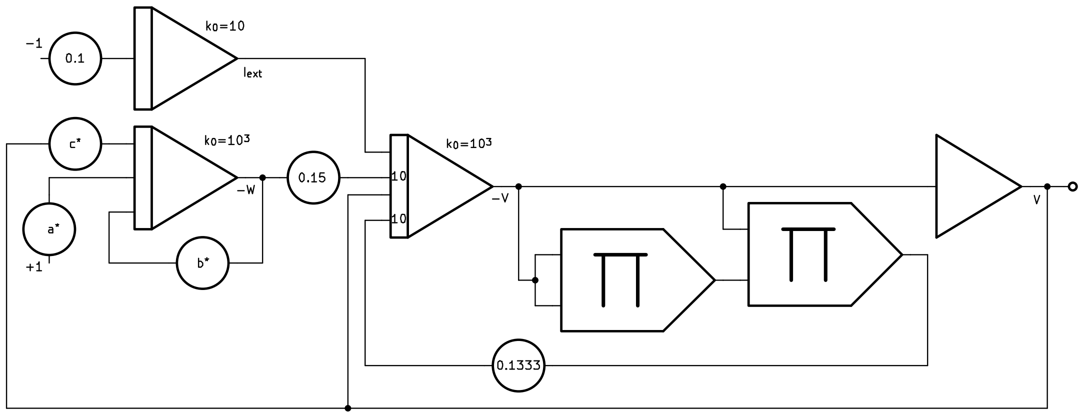

Figure 1: Analog computer program for the FitzHugh-Nagumo model

This, too, requires a change to equation 2 to compensate for λ_W:

$$\dot{\widehat{V}} = \widehat{V} - \frac{4}{3}\widehat{V}^3 - \frac{3}{2}\widehat{W} + I_{\text{ext}}. \tag{3}$$

Introducing scaled parameters $a^* = \frac{1}{3}\tau^{-1}a = 0.0186$, $b^* = \tau^{-1}b = 0.064$, and $c^* = \frac{2}{3}\tau^{-1} = 0.0533$ finally results in the second scaled equation

$$\dot{\widehat{W}} = c^*\widehat{V} + a^* - b^*\widehat{W}. \tag{4}$$

The equations (3) and (4) can be implemented directly on an analog computer such as THE ANALOG THING.[5] Using an additional integrator, the stimulus current I_{ext} can now be automatically varied as a linear ramp from 0 to 1 showing the overall behavior of the system. Figure 1 shows the resulting analog computer program. Its actual implementation on THE ANALOG THING can be seen in figure 2.

The result of one run with $0 \leq I_{\text{ext}} \leq 1$ is shown in figure 3. For values of I_{ext} smaller than a certain threshold no spiking occurs. Spiking starts in the third division of the screen shot and ends when I_{ext} exceeds a second much higher threshold.

[5]See `https://the-analog-thing.org`, accessed 30.07.2026.

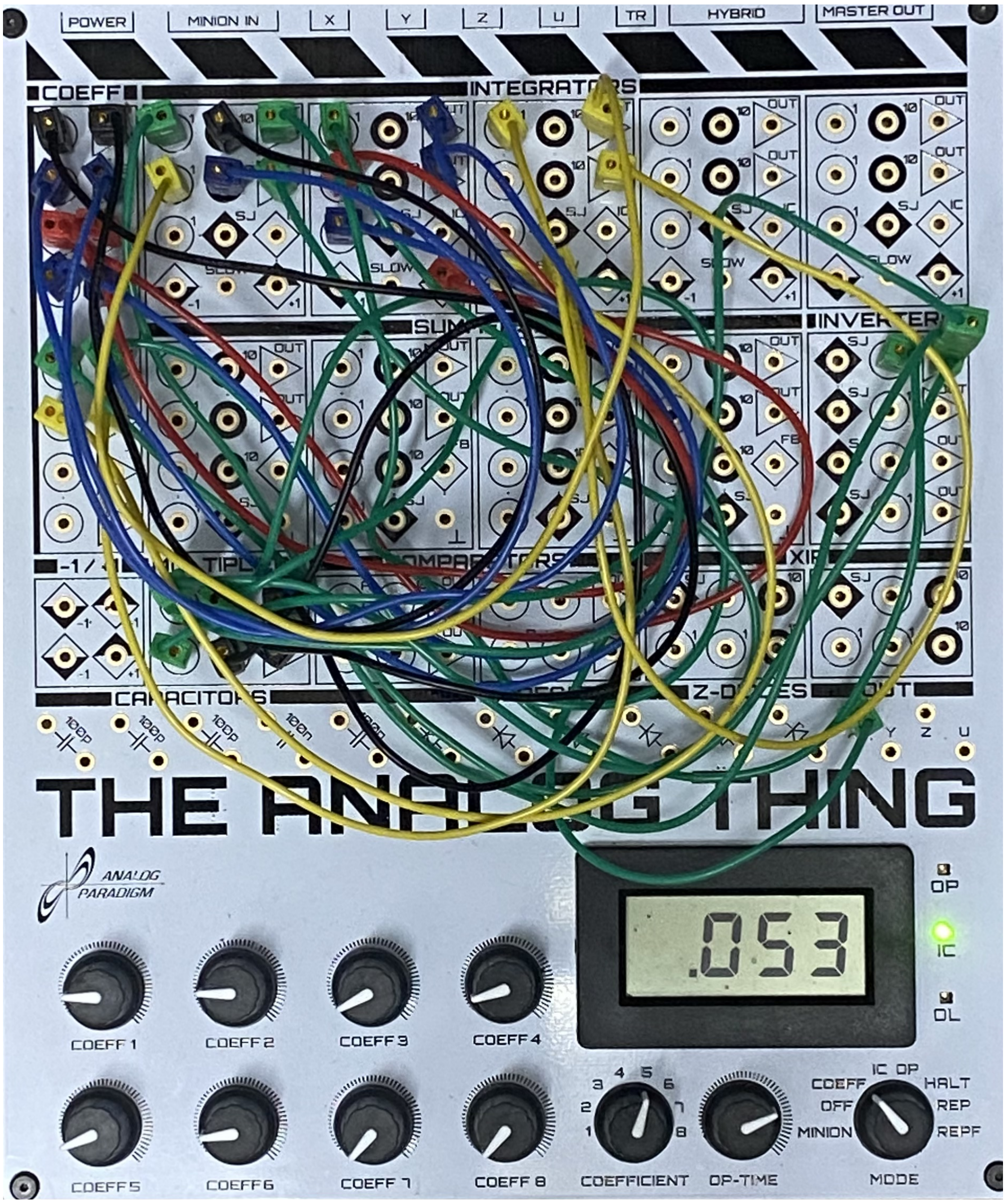

Figure 2: Implementation of the FitzHugh-Nagumo model on THE ANALOG THING

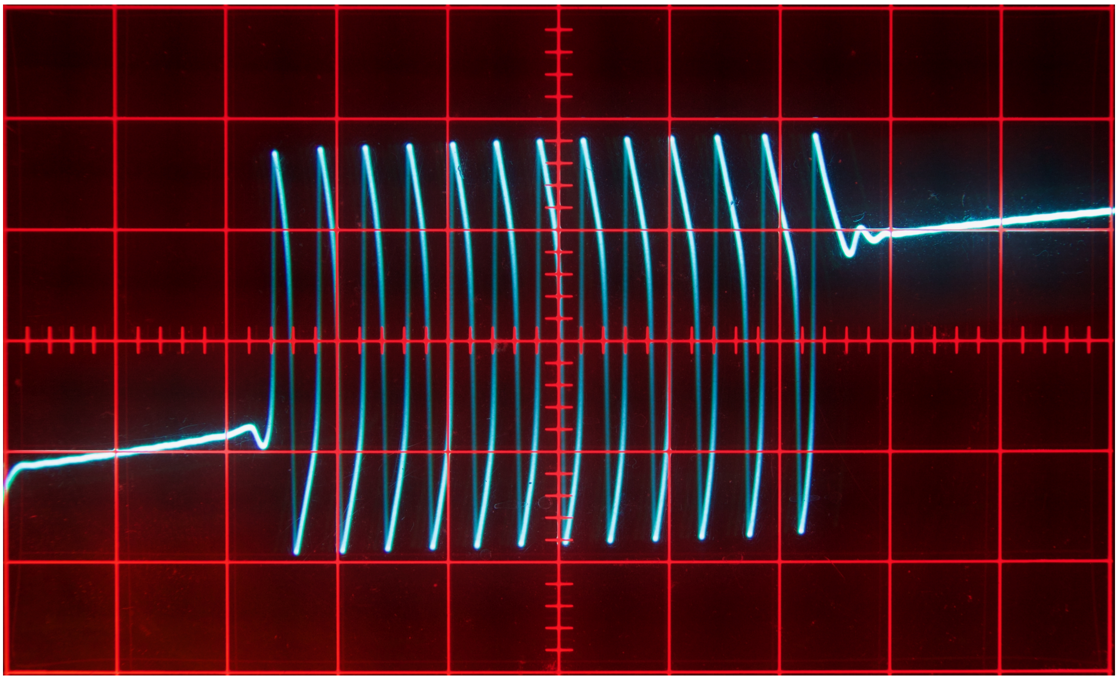

Figure 3: Typical result obtained by the FitzHugh-Nagumo model, 5 V/div vertical, 100 ms/div horizontal, $0 \leq I_{\text{ext}} \leq 1$

Happy analog computing!

References

[Izhikevich 2007] Eugene M. Izhikevich, *Dynamical Systems in Neuroscience: The Geometry of Excitability and Bursting*, The MIT Press, 2007

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[Raman et al. 2021] Indira M. Raman, David L. Ferster, *The Annotated HODGKIN & HUXLEY – A Reader's Guide*, Princeton University Press, 2021